ENHANCING MATH LESSONS WITH A BLENDED TECH-PEDAGOGY APPROACH

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ABSTRACT

Undergraduate mathematics courses play a vital role for students aiming for professions in the domain of science, technology, engineering, and mathematics fields. However, many students find mathematics courses challenging and intimidating, leading to high dropout rates and not up to mark performance. This paper reviews pedagogical strategies for effective teaching of undergraduate mathematical sciences courses, based on current research and best practices. These strategies include the use of technology, clear and concise explanations that encourage deep understanding. By implementing these strategies, course instructors can create a positive learning environment that supports student success in undergraduate courses. To accomplish this, Mathematica, Wolfram Alpha, R, and Matlab tools are tested on a set of problems. The novelty in using Computer Algebra System (CAS) tools lies in the advancement of machine learning algorithms, which have resulted in the advancement of more sophisticated and easy-to-use CAS interfaces. Students can find the opportunity of numerous experiments that provide some insights of problem solving methods. Further, the use of these computer applications provide meaningful understanding for the student, and an opportunity to verify the manual calculations. Mathematica and Wolfram Alpha seem to provide better results with respect to results accuracy and plots of useful functions. This study would provide a new challenge to *both teachers as well as undergraduate and research students of these disciplines. Moreover, the paper highlights how these tools provide students with opportunities to verify manual calculations and explore different solution methods, fostering deeper understanding and critical thinking.*

KEYWORDS
Computer algebra system, Mathematica, Wolfram Alpha, R and Matlab

INTRODUCTION

A strong foundation in mathematics is offered by undergraduate courses for those interested in careers in science, technology, engineering, and mathematics (Pöhler, B, 2017; Borovik, A., 2023; Browne, E. C. 2022). However, a lot of students find math classes hard and scary, which results in low performance and high dropout rates. This paper reviews strategies for effective teaching of undergraduate mathematics, based on current research and best practices (Hypolite, S., & Myll, A. 2021; Vavilov, N. 2020). These strategies include use of technology, clear and concise explanations, and assessment that encourages deep understanding. By implementing these strategies, course instructors can create a positive learning environment that supports student success in undergraduate mathematics courses. To accomplish this, Mathematica, Wolfram Alpha, R, and Matlab tools are tested on a set of problems. The novelty in using Computer Algebra System (CAS) tools lies in the progression of machine learning algorithms, which have led to the advancement of more sophisticated and user-friendly CAS interfaces (Rocklin, M., & Terrel, A. R. 2012; Meurer, A. 2017). It is used in many fields, including engineering, physics, finance, and biology, to name just a few.

For the last three decades, the use of technology in classroom teaching has been in practice in most of the educational institutions of the world, which in general, strengthens students' learning process (Abdullayeva, M. 2021). In particular, the use of technology in most of the mathematical courses helps to enhance concepts of mathematical objects, and analytical reasoning in students (Hamid, H. 2021; Ardiç, M. A. 2021). The symbolic package is used to explain students' vector spaces (Açıkyıldız, G., & Kösa, T. 2021). Several symbolic packages for mathematical sciences subjects are available such as Macsyma, Python, Mathematica, Matlab, Mathcad, or Maple for education as well as researchn. CAS tools are also used for the solution of PDEs by applying an appropriate generalized coordinate system (Opanasenko, S. 2017). In some universities, the CAS tool is used to teach linear algebra courses (Rensaa, R. J. 2020). It is important to note that engineering students achieve meaningful mathematics learning if they are provided hands-on actions (Awang, T. S., & Zakaria, E. 2012; Nazar, M. et al., 2022).

Real-life problems, related to economic, industrial, and engineering disciplines are generally modeled with linear and/or nonlinear programming formulations. In many situations the objective function and / or some of the constraints are non-linear in nature. Optimization methods also play a crucial role in many problem-solving activities in machine learning, data science, and statistics disciplines. The implementation of these approaches depends on specific optimization techniques tailored exclusively for a particular application. Algorithms related to convex optimization are widely used in modern solvers to obtain the solutions to the given problems (Theußl, S. et al., 2020). A few R utilities will be used, which helps to formulate and solve optimization problems consistently, and is capable of modeling linear, and general nonlinear optimization problem (Liu, X., & Ma, Y. Q. 2023). Sometimes CAS tool fails to produce the correct solution to the problem as extra information is required to obtain a feasible solution. So, a comparative analysis of results from more than one computer application is helpful to get the correct solution to the problem. Therefore, it is crucial to ensure that mathematics education aligns with advancements in science and technology (Reijers, S. A. 2019). Nonlinear built-in Mathematica functions are useful both for exact and numeric solutions of problems (Harris, J., & Al-Bataineh, A. 2015). In imparting online education, the Wolfram Alpha package is a very useful tool to do various scientific and engineering calculations not only for researchers but also for instructors and studentsn (Глушак, O. M. et al., 2019).

This study provides a mathematical formulation of simple word problems and solutions to algebraic problems, solves a few business and engineering problems with commands available in Wolfram Alpha, Mathematica, R, and Matlab and demonstrates 2D and 3D graphs of some important functions. Additionally, this study also focuses on enhancing students' comprehension of abstract mathematical concepts through computational tools, encouraging active learning, and promoting experimentation for problem-solving. Research methodology include criteria such as: ease of use of these Matlab, Mathematica, and R in teaching. Computational capabilities like accuracy, speed, and versatility in solving mathematics problems. Use of visualization tools Educational resources example problems are given below:

LITERATURE REVIEW

Problem Solutions with Matlab and Mathematica

In this section, we solve a few cases of linear programming and calculus problems with Mathematica, and Matlab computer applications. Some problems can be solved exactly, while others are solved numerically using different methods available in CAS tools. The use of CAS tools during class teaching seems to be very useful both for hand calculation, and doing comparisons of solution accuracy. Following are the examples.

The mathematical form of a linear programming problem is given by.
Min $f_1x_1 + f_2x_2 +\ldots+ f_nx_n$
X s. t.
$a_{11}x_1 + a_{12}x_2 +\ldots+ a_{1n}x_n = b_1$
$a_{21}x_1 + a_{22}x_2 +\ldots+ a_{2n}x_n = b_2$
$a_{m1}x_1 + a_{m2}x_2 +\ldots+ a_{mn}x_n = b_m$
$x_j \geq 0,\ j=1, 2,\ldots, n$ (1)

In matrix notation it can be written as follows (Luenberger, D. G., & Ye, Y. 1984).
$Min\ f^Tx$ subject to $Ax = b,\quad x \geq 0$ (2)

Here, x represents a vector of n variables, and $Ax = b$ denotes linear equality constraints of nonnegative variables. In example 1, a linear optimization problem is solved.

Case 1: ***Solve the Linear Programming problem***
Maximize $F = 2x + y + 3z$
Subject to: $x + 2y + z \leq 12$ (4)
$2x + y + z \leq 20$ (5)
$x + y + 2z \leq 20$ (6)
$x, y, \quad z \geq 0$ (7)

Matlab solution

```
>> f=[2 1 3]'; A=[1 2 1; 2 1 1; 1 1 2]; b=[12; 20;20]; Ae=[]; Be=[]; lb=[0 0 0]';
>> [x fval] = linprog( -f, A, b, [], [], lb );
```

Optimal solution is found.

```
>> x=[4,0,8]
>>fval = -32
```

Negative sign with function value is neglected as the problem was of function maximization.
If we try to find out the solution without using a lower bound, Matlab will not give the solution.

```
>> [x fval] = linprog( -f, A, b, [], [], [] );
```

Message produced by the Matlab is: *Problem is unbounded*.

So, we observe that Matlab is unable to solve this problem if a lower bound is not provided with the *linprog* command. On the other hand, the Mathematica *Linear Programming[ ]* command gives the correct solution if we use only *f*, A, and *b*. Similarly, Mathematica *Maximize[ ]* command also gives the same result (Bhadani,

R. 2021). It is important to note that students now feel confident while using more than one CAS tools to verify the results of the problem. Now, the same problem is solved using Mathematica.

Solution from Mathematica

Mathematica assumes the format **A.x≥0,** therefore, inequalities (4-6) are multiplied by -1.

[In]:=*LinearProgramming* [{-2,-1,-3},{{-1,-2,-1},{-2,-1,-1},{-1,-1,-2}},{-12,-20,-20}]
[Out]:= {4,0,8}
And corresponding max-value of the objective function will be 32

Here is an example of how to perform differentiation using both Matlab and Mathematica.

Case 2: ***Find the derivative of*** $f(x) = x^3 + 2x^2 + x$

In Matlab: We enter.

```
>> syms x; f = x^3 + 2*x^2 + x;          % define the function to differentiate
>> df = diff(f,x);                        % differentiate the function with respect to x
>> disp( df);                             % display the result
```

The output will be:
3*x^2 + 4*x + 1.

In Mathematica: We enter.

```
f[x_] := x^3 + 2*x^2 + x;
(* define the function to differentiate *)

df[x_] := D[f[x],x];
(* differentiate the function with respect to x *)
df[x]  (* display the result *)
```

The output will be: 1+ 4 x + 3 x^2.
It can be observed that the syntax for defining and differentiating functions is slightly different between Matlab and Mathematica, but the overall process is very similar. Both programs use symbolic variables and functions to perform differentiation. The next section elaborates on some cases related to business and engineering disciplines (Venkataraman, P. 2009).

Business and Engineering Problems Solutions

CAS software packages such as Mathematica, Maple, R, and Matlab, etc., have built-in tools for performing regression analysis, and other statistical utilities in addition to complex mathematical problem-solving approaches. The CAS includes integrated algorithms for addressing stochastic issues, including Monte Carlo simulations and optimization techniques. These tools can likewise be applied to solve problems in business and engineering.

Case 3: Consider the design of a speed reducer

The process of designing a speed reducer demands choosing suitable components to produce a specific output speed and torque based on a certain input speed and torque. This means that several engineering factors must be considered, such as:

i) first you need to set up the input speed and torque in accordance with the desired output speed and torque.

ii) Secondly, select the perfect gear ratio that is, the relationship between the number of turns of the input and the output axle turns.

iii) The next step is to choose the right type of gear. There are various options such as inter alia, spur, helical, bevel, and worm gears. Depending on the application's requirements, efficiency and life expectancy, the appropriate gear must be chosen.

iv) Make sure to choose the relevant materials after careful consideration so that they can withstand the forces and loads they are supposed to bear.

v) To ensure longevity of speed reducer, it is important to keep it well lubricated. This will protect the speed reducer and avoid corrosion.

vi) Make a proper plan and analyse all the costs associated with the manufacturing of a speed reducer.

Using specialized software such as Mathematica, Matlab and R, researchers obtained estimated solutions for speed reducer designs (Sadollah, A. et al., 2013; Papa Christodoulou, A. et al., 2013). The calculations carried out using Mathematica, Matlab, and R were compared to the target literary reference solution. The objective function value from the document's accuracy was compared with the solution of 2994.48 for the speed reducer design case problem.

Table 1: ***Performance of Mathematica, Matlab, and R.***

Package	Objective function	Relative error

Mathematica	2996.35	6.24×10^{-4}
Matlab	2889.97	3.49×10^{-2}
R	2721.89	9.10×10^{-2}

Overall, the design of a speed reducer requires careful consideration of a number of factors, including the input and output specifications, gear ratio, gear type, material selection, lubrication, and manufacturing considerations. A successful design will result in a speed reducer that meets the application requirements while providing efficient and reliable performance.

Now, we consider an economic problem.

Case 4: Here is an example of an economics problem that can be solved using a CAS tool like Mathematica

Suppose a firm is producing a certain product, and the cost function of producing Q units is given by $C(Q) = 100 + 5Q + 0.1Q^2$. The firm sells the product at a price of $10 per unit, and the demand function is given by P(Q) = 200 - 2Q.

The challenge is to determine the level of output that maximizes profit for the company.

To solve this problem using a CAS, we can start by defining the cost function and the demand function in Mathematica:

```
C[Q_] := 100 + 5Q + 0.1Q^2;
P[Q_] := 200 - 2Q;
Q = (P'[Q]*C''[Q] + P''[Q]*C'[Q])/(2*P''[Q]);
Q = (P'[Q]*C''[Q] + P''[Q]*C'[Q])/(2*P''[Q]);
Q /. {P'[Q] -> -2, P''[Q] -> 0, C'[Q] -> 5 + 0.2Q, C''[Q] -> 0.2}
```

The output is given by: 25

```
Profit = P(Q) * Q - C(Q)
Profit /. Q -> 25
Output: 187.5
```

This part showcased examples from both engineering and business fields. These tools can address issues across nearly every area of life, as long as their mathematical models can be developed. Subsequently, we will apply these tools to generate 2D and 3D representations of certain functions.

Visualizing Functions with Mathematica and Wolfram Alpha

Mathematica, Maple, and Wolfram alpha are computer Algebra Systems (CAS) that can created both 2D and 3D graphs of functions that too instantly. The systems are highly advanced and use complex algorithms and mathematical equations to produce accurate representations of functions in three-dimensional space. For an accurate representation of the graph it is highly important to choose the correct function and input parameters in the CAS tool. Once the data is inserted, CAS can generate the 3D graph within seconds. In contrast to traditional methods of graphing in three dimensions, CAS is much more efficient and less time-consuming, producing precise graphs. This is extremely beneficial in the areas of mathematics, physics and engineering where three-dimensional visuals of functions help a lot in problem solving. Mathematica and Wolfram Alpha are two highly specialized programs that can produce practical graphical representations of functions some examples are given below:

Case 5: Obtain plot of $f(x) = x^2$. We can input the subsequent command in Mathematica or Wolfram Alpha.

Plot[x^2 , {x, -10,10}]

It will produce the following graph.

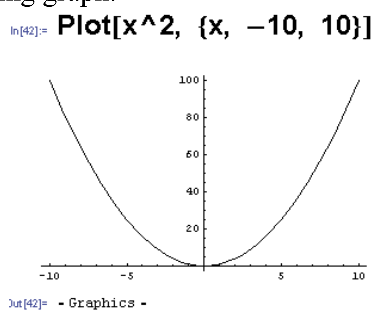

Figure 1: ***Plot of parabola using Mathematica.***

Plot({x^2 , Sin[x]}, {x,-5,5})

It will produce the following plot. This produces two graph in single sheet.

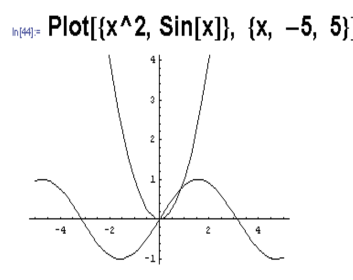

Figure 2: ***Two curves on a single graph.***

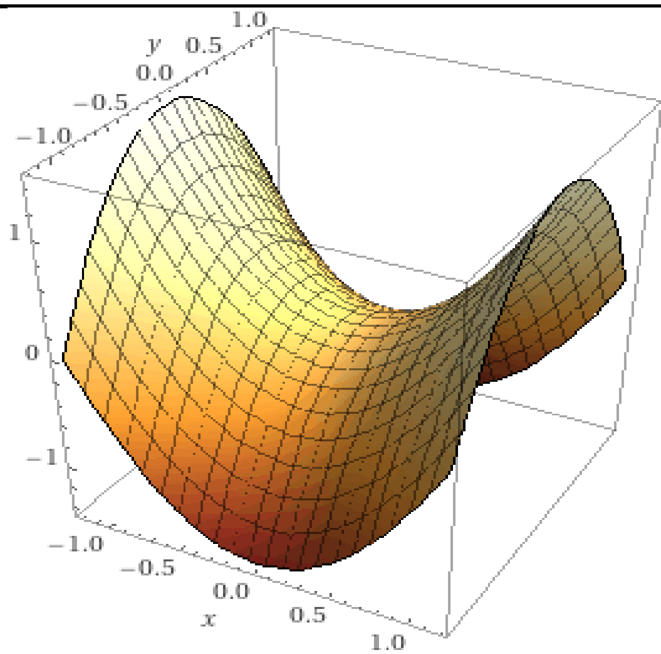

Figure 3. 3D plot of : *f*[*x*,*y*]= x^2-y^2 using Wolfram alpha.

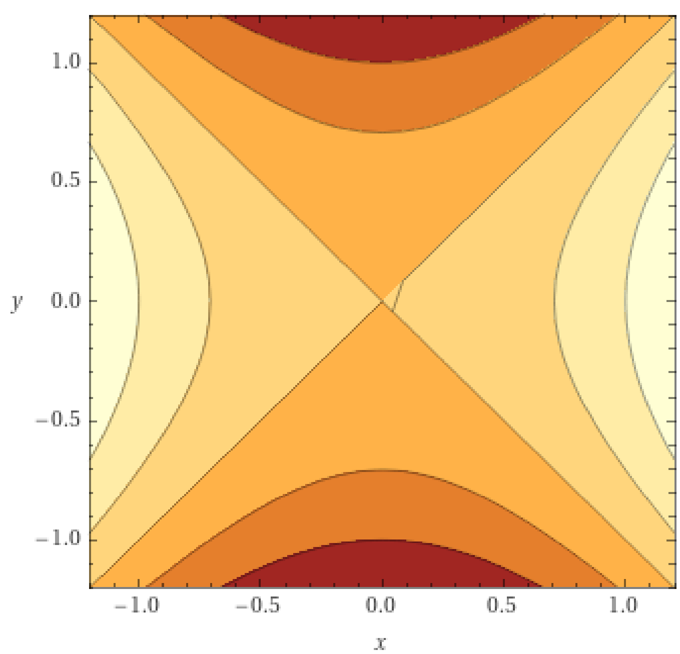

Figure 4. Contour plot of *f*[*x*,*y*] = x^2-y^2 using Wolfram alpha.

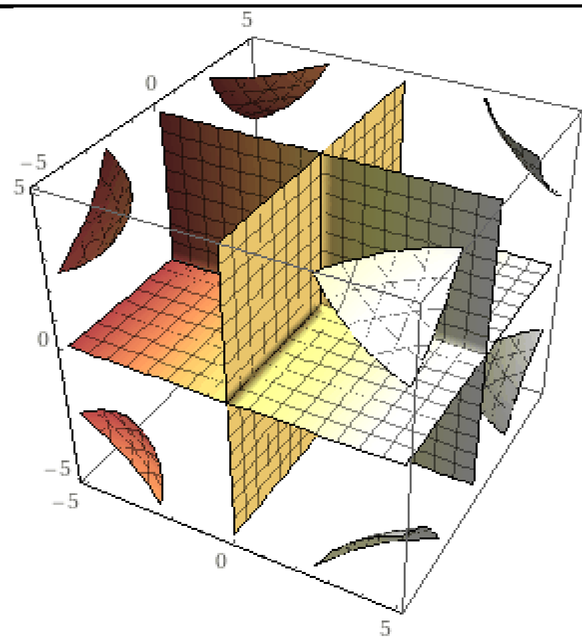

Figure 5: ***3D plot of the function f(x,y,z) = xyz.***

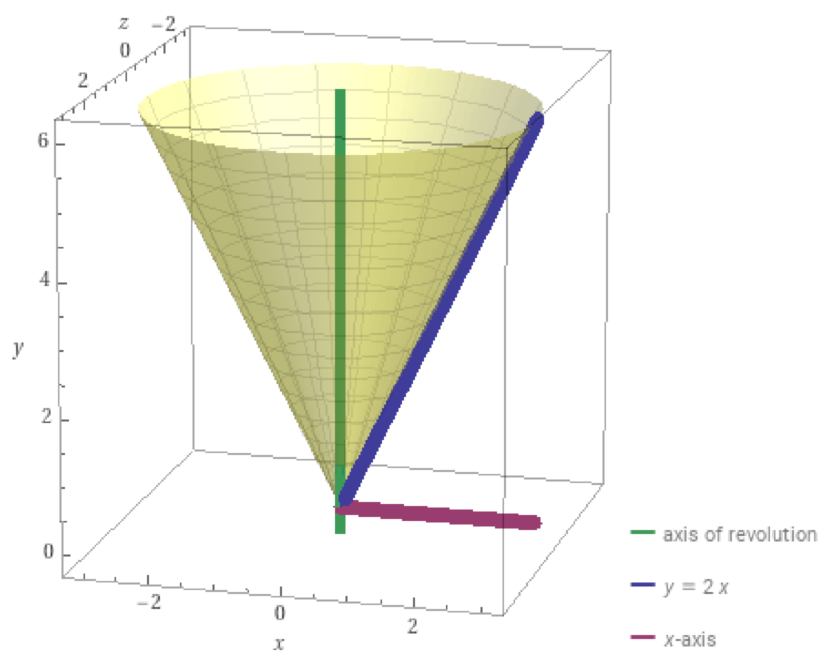

Figure 6: ***3D plot generated from Wolfram alpha.***

Figure 6 is obtained by entering the following command in Wolfram Alpha (y=2x, 0<x<3 Rotate about y-axis).

RESEARCH OBJECTIVES

1. To evaluate the usability and user-friendliness of Mathematica, Matlab, and R in the context of teaching mathematics courses, focusing on the ease of use for both instructors and students.
2. To compare the computational efficiency of Mathematica, Matlab, and R in solving Calculus problems, with respect to processing time and accuracy.
3. To assess the visualization capabilities of each software tool, analyzing how effectively Mathematica, Matlab, and R present Calculus concepts through graphical representations of functions, derivatives, integrals, and 3D plots.

RESEARCH METHODOLOGY

Research methodology include criteria such as: ease of use of these Matlab, Mathematica, and R in teaching. Computational capabilities like accuracy, speed, and versatility in solving mathematics problems.

DISCUSSION

Performance of *Matlab*, *Mathematica, Wolfram Alpha*, and R has been elaborated in this study by taking several problems from science and engineering disciplines. Comparison of solution outputs reveals that *Mathematica* is, in general, better for accurate calculations. One of the major benefits of using a CAS tool is that it can perform complex calculations and manipulations of mathematical expressions in a fraction of the time it would take a human to do so. This can save researchers and educators a significant amount of time and effort, allowing them to focus on more critical aspects of their work. Another advantage of using CAS tools is that they can provide visual representations of mathematical concepts, making them more accessible to students and researchers who may struggle with abstract concepts. This can be especially helpful in fields such as physics, engineering, and economics, where complex mathematical models are used to describe real-world phenomena. According to the study's findings, according to the recent research, it has been found out that Mathematica and Wolfram Alpha produces higher-quality 2D and 3D plots of various functions (as shown in fig. 1-6). Wolfram Alpha created a three-dimensional graph using the simple command (y=2x, 0<x<3) to revolve around the Y-axis. This allows students to track the solution process through better understanding of the problem‘s solution.

CAS is a powerful tool that can be used to solve complex mathematical problems accurately that too in very less time. CAS software like Mathematica, Wolfram Alpha, and Matlab can handle complicated mathematical problems, visualise the data for a better understanding of the problem, and come up with exact solutions and resolve symbolic equations. This feature has the potential to greatly expedite the modeling process and reveal insights that may not be immediately apparent when using only numerical methods. Using CAS tools can speed up the problem-solving process and display the solution of the problem in depth. This is extremely beneficial for scientists, engineers, and academics professionals. CAS is very useful in education as well because it carries the features for evaluating and interpreting mathematical data and producing fine clear results. With the use of these potent tools, analysts and researchers can better comprehend complex mathematical problems and use the information they gather to inform their decisions. In the upcoming correspondence, we will examine the ways in which Matlab, Mathematica, and R can be applied to solve complex real-world issues (Ogle, D. H. 2018; Xue, D. 2020).

RECOMMENDATIONS

1. Regardless of which software is chosen, it is essential to provide thorough training for both students and instructors to reduce the learning curve and increase the efficiency of using these tools.
2. Online forums, discussion groups, and user communities for each tool can be valuable resources, and their integration into the learning process should be encouraged.
3. Depending on the teaching style, Matlab may be better suited for instructors who prefer a more computational, hands-on approach that emphasizes coding and numerical analysis.
4. Mathematica is ideal for instructors who prefer to teach theoretical aspects of Calculus and want to showcase both symbolic manipulation and visualizations.
5. If the course emphasizes practical problem-solving or real-world applications, Matlab is highly recommended due to its widespread use in engineering, physics, and applied mathematics.

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